

## ● Letters to the Editor-in-Chief

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### A Note from the Editor-in-Chief:

Until now, the publication in the journal of a Letter to the Editor-in-Chief has been quite a rare event. When the Editor receives such a Letter, if it relates to an article previously published in the journal, a copy is sent to the corresponding author concerned. This provides the opportunity for a response to be published at the same time as the original Letter. Apart from making sure that nothing obviously inappropriate is published, the Editor does not intervene in this process. What is published as a Letter is not subjected to peer review. The justification is that such a published Letter

is itself an integral part of the peer review process. In principle, the process can go through several cycles, with each pair of published Letters leading, in a subsequent issue of the journal, to a further pair of Letters, until the Editor brings the correspondence to an end.

The journal does not currently have a policy concerning the length of an individual Letter. Notice is now given, however, that Letters that would occupy more than two printed pages in the journal will not normally be accepted for publication in the future.

P.N.T. WELLS

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## COLOR FLOW MAPPING

### To the Editor-in-Chief:

This letter pertains to the review article “Color flow mapping” by Ferrara and DeAngelis (1997), which was published in this journal (1997;23:321–345). The topic is timely and important; however, while reviewing the alternative velocity estimation techniques in the literature, the authors unfortunately have omitted many important contributions to the field.

This letter provides a functional classification of estimation techniques, along with the inclusion of important articles missing from the review article to provide the readers with a more complete picture of the field. We wish to point out particularly lucid and important published work including those by Burns (1987), Jensen (1996), Magnin (1986, 1987), Kremkau (1990, 1991), Vaitkus and Cobbold (1988) and Vaitkus (1995), to name a few, as well as comment on the “butterfly search” technique (Alam 1996; Alam and Parker 1995, 1996) and the other techniques within the same functional classification, which were surprisingly absent from the review article.

Velocity estimation methods can be grouped into three principal categories, primarily based on the signal models: 1) the frequency/phase (Doppler) methods, 2) the time domain methods and 3) the multiple-burst (tracking) methods. The methods are sometimes inexactly classified into narrow-band and wide-band methods.

1) **The frequency/phase (Doppler) methods.** If there is target motion between transducer firings, the echoes will shift. If the echo is sampled at a fixed depth over repeated firings, then this shift will also appear as a change of phase in the returned echo from the sample volume. The rate of change of phase (or the frequency) of the signals at the fixed sample volume for progressive transducer bursts will be proportional to the tissue velocity as long as the tissue does not move by more than a quarter of the wavelength between transducer firings (otherwise, aliasing occurs). Various esti-

mation methods can be used to determine this frequency and, hence, the velocity. The frequency domain methods use relatively narrow-band pulses and typically use eight or more transducer bursts. No tracking of the tissue movement is performed. The breakthrough implementation for color flow imaging came from this group (Kasai et al. 1985). The velocity is computed from the phase of the autocorrelation of sampled complex envelopes at unity lag. Liu et al. (1991) described a modified autocorrelation method that incorporates a spatial vector-averaging technique. Computer simulations showed superior performance in a noisy environment or with short data sets. Barber et al. (1985) proposed a time domain processing scheme of the quadrature components to detect the Doppler frequency, which is similar to the autocorrelation estimator. Li (1995) introduced a frequency domain autocorrelation technique for blood velocity estimator. van Leeuwen et al. (1986) discussed four spectral estimators, namely, the phase detector, zero-crossing detector, instantaneous frequency detector and autocorrelator, the autocorrelator performing the best in the simulations. Four spectral estimation techniques have been discussed by Vaitkus and Cobbold (1988) and Vaitkus et al. (1988), specifically, periodogram, moving average (MA) model, autoregressive (AR) model (Ahn and Park 1991; Kuc and Li 1985; Loupas and McDicken 1990) and autoregressive-moving average (ARMA) model. MA, AR and ARMA methods were found to outperform the fast fourier transform (FFT)-based periodograms at the expense of more computation. However, a major concern for the AR and ARMA techniques is finding the optimal number of model parameters. Forsberg (1991) found that no single set of model orders was capable of producing consistent spectral estimates throughout the cardiac cycle when applying the singular value decomposition (SVD)-ARMA algorithm to *in vivo* Doppler signals. Magnin (1986) wrote a classical tutorial article on Doppler ultra-

sound that also discussed many intricate technical issues involved with velocity estimation. Burns (1987) and Magnin (1987) provide excellent overviews of conventional Doppler methods. Jensen (1996) and Jones (1993) also provide a comprehensive overview of the estimation of blood velocity with ultrasound. Kremkau (1990, 1991) and Wells (1994) provide excellent summaries of color flow applications.

2) **The time domain methods.** The relative shift in echoes due to target movement between two successive firings also can be used to estimate flow. Two groups independently implemented cross-correlation search to estimate this shift (Bonnetfous and Pesque 1986; Foster et al. 1990). Because the correlation search is computationally intensive, a one-bit version has been implemented commercially for real-time operation (Bonnetfous et al. 1986; Rickey et al. 1992). To reduce the computational complexity for the correlation search, a SAD algorithm has been proposed (Bohs and Trahey 1991). Time domain search methods generally use wide-band pulses and do not suffer from aliasing. de Jong et al. (1990) developed a narrow-band method that computes the correlation coefficient function only at five points in the vicinity of the maximum, and uses an interpolation algorithm to evaluate the location of correlation maximum from these points. This method requires significantly less computation compared to correlation search; however, it does not work well with large bandwidth signals, and it suffers from aliasing. This technique has been evaluated in an experimental set-up with a rotating agar disk containing scattering particles (de Jong et al. 1991). Jensen (1991, 1993, 1994) discussed the performance, limitations and artifacts associated with the time domain correlation methods for tissue motion estimation. A comprehensive review of the time domain methods can be found in Hein and O'Brien (1993).

3) **Multiple-burst (tracking) methods.** These methods are based on both the phase change and the relative shift in the echoes. Recall that the frequency/phase domain methods use the change of phase over many transducer bursts from only a fixed sample volume, whereas the time domain methods, at least in the simplest implementation, try to estimate the tissue movement through the time shift between two successive pairwise transducer bursts. However, the new tracking methods try to track the target movement using various algorithms through many transducer bursts. In this model, the radio frequency (RF) echoes due to many transducer bursts are stacked together to form a two-dimensional (2D) array ("slow time": transducer burst index, "fast time": depth). The 2D array thus created may be processed in a variety of ways to obtain a flow estimate. These are generally wide-band methods, but some suffer from aliasing. There have been many contribution to this area; however, the authors of the review failed to discuss any work in this area other than their own. To the best of our knowledge, Wilson (1991) was among the pioneers in this group of methods. He proposed using the 2D Fourier transform on the 2D RF array. It was shown that the 2D Fourier transform would be nonzero only along radial line segments on the frequency plane whose slope is proportional to the scatterer velocity. A method to overcome the associated frequency aliasing problem also was discussed. A different matched filter approach referred to as the wide-band maximum likelihood estimator (WMLE) was developed by Ferrara and Algazi (1991). The likelihood for velocity  $v$  was given by:

$$l(v) = \left| \sum_n \int_{-\infty}^{\infty} \tilde{r}(t) \tilde{s}^* \left( t - 2\frac{d}{c} - nT \left[ 1 + 2\frac{v}{c} \right] \right) \exp \left( j2\omega_0 \frac{v}{c} t \right) dt \right|^2, \quad (1)$$

where  $d$  is the initial distance of the object from the transducer,  $\omega_0$  is the angular center frequency,  $n$  is the index for the repeated transducer bursts (slow-time index),  $T$  is the pulse repetition period,  $\tilde{r}(t)$  is the complex envelope of the received echo and  $\tilde{s}(\bullet)$  is a delayed version of the complex envelope of the transmitted pulse (including effects of signal propagation and scattering). The factor of two in the expression comes from the round-trip travel of the wave. The time-axis origin is reset each time the transducer fires. The mean velocity can be computed from the ratio of the first moment of  $l(v)$  and the zeroth moment of  $l(v)$ . The WMLE later was modified slightly since the exponential term varies slowly with respect to  $t$ , and thus can be taken outside of the integral (Ferrara et al. 1996):

$$l(v) = \left| \sum_n \exp \left( j2\omega_0 \frac{v}{c} nT \right) \int_{-\infty}^{\infty} \tilde{r}(t) \tilde{s}^* \left( t - 2\frac{d}{c} - nT \left[ 1 + 2\frac{v}{c} \right] \right) dt \right|^2. \quad (2)$$

Alam and Parker (1995) developed the "butterfly search" for envelope, RF or complex envelope signals from a deterministic analysis (the latter derived using Schwartz's inequality), based on the common signal model used in all methods in this group. In this method, the complex envelope is sampled on different delay trajectories (butterfly lines) on slow time-fast time space. Each butterfly line is associated with a unique frequency. The butterfly-sampled complex envelope is examined for the power at the frequency corresponding to that butterfly line, normalized by the total power. On each butterfly line, the following ratio is evaluated:

$$L(v) = \frac{\left| \sum \tilde{r}_{Bv}[n] e^{j2\omega_0 \frac{v}{c} T} \right|^2}{\sum |\tilde{r}_{Bv}[n]|^2} \quad (3)$$

where  $\tilde{r}_{Bv}[n] = \tilde{r}(n, t) \delta \left( t - 2\frac{d}{c} + 2n\frac{v}{c} T \right)$  and denotes the resampling of the complex envelope along the butterfly line for velocity  $v$  at depth  $d$ . The velocity is generally computed from the maximum of  $L(v)$ . The mean velocity can be estimated from the first moment of  $L(v)$  divided by the zeroth moment of  $L(v)$ . A hardware implementation is shown in Fig. 1.

They later followed their work with two techniques to reduce the computational complexity of the method and provide sampler hardware implementation (Alam and Parker

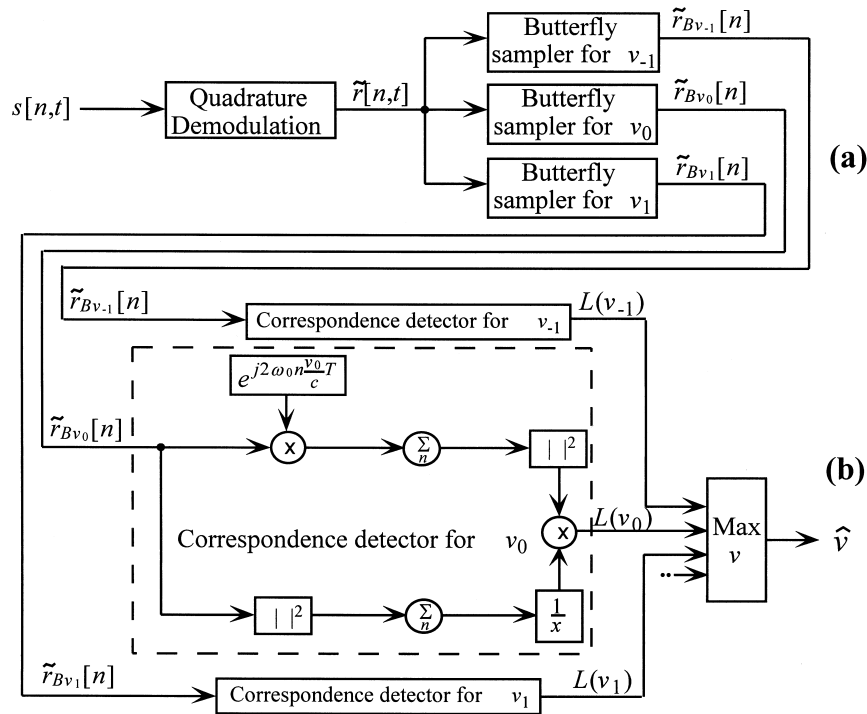


Fig. 1. Schematic hardware diagram for quadrature butterfly search. (a) Butterfly sampling of complex envelope; (b) processing  $\tilde{r}_{Bv}[n]$  to estimate  $v$ .

1996). In one method, the multiplication of the complex envelope by the complex exponential after the butterfly sampling is replaced by multiplying the complex envelope with an RF complex exponential prior to the butterfly-sampling step. Furthermore, this was shown to be equivalent to using the analytic RF signal. The butterfly search on quadrature components has shown much superior performance in low signal-to-noise ratio (SNR) conditions (typical in blood velocity estimation due to low scattering from blood) compared to the earlier methods (Alam and Parker 1995, Alam 1996). A detailed analysis and comparison with the current methods are given in Alam (1996).

Under a number of specific assumptions and modifications, the WMLE in eqn (2) could be modified into the form of the butterfly search on quadrature components, as described in eqn (3). The key modifications that need to be made include: 1) the matched filter is changed to a short rectangular window (the key-matched filtering operation in WMLE is removed); 2) the higher-order term involving  $v$  is dropped from the argument of the complex envelope; and 3) an *ad hoc* denominator normalization term is included. These key modifications were demonstrated by Alam and Parker (1995). Following publication of the article by Alam and Parker (1995), Ferrara et al. (1996) included a denominator term to the WMLE only at its maximum in an attempt to remove the velocity estimation for the noise that remains after the clutter rejection filter, and termed it the likelihood magnitude:

$$\begin{aligned}
 lm(v)|_{v=v_m} &= \left| \sum_n \exp\left(j2\omega_0 \frac{v_m}{c} nT\right) \int_{-\infty}^{\infty} \tilde{r}(t) \tilde{r}^* \left( t - 2\frac{d}{c} - nT \left[ 1 + 2\frac{v_m}{c} \right] \right) dt \right|^2 \\
 &= \frac{\left( \sum_n \left| \int_{-\infty}^{\infty} \tilde{r}(t) \tilde{r}^* \left( t - 2\frac{d}{c} - nT \left[ 1 + 2\frac{v_m}{c} \right] \right) dt \right|^2 \right)}{\left( \sum_n \left| \int_{-\infty}^{\infty} \tilde{r}(t) \tilde{r}^* \left( t - 2\frac{d}{c} - nT \left[ 1 + 2\frac{v_m}{c} \right] \right) dt \right|^2 \right)}. \quad (4)
 \end{aligned}$$

Note that the butterfly normalization term in eqn (3) has a different form from that in eqn (4), as eqn. (3) was derived using Schwartz's inequality. Beyond the distinctions in mathematical terms, there are four important differences between the butterfly search and WMLE:

- (1) The butterfly search is a family of techniques that apply to the envelope, or RF, or complex envelope.
- (2) The butterfly search is based on a deterministic signal analysis, and the implementation for the complex envelope derives directly from Schwartz's inequality.
- (3) The butterfly search can be related to the discrete Radon transform (Durrani and Bisset 1984).
- (4) The butterfly search has a direct hardware implementation using elementary digital operations.

Independently published work has discussed the performance and limitations of the WMLE (Vaitkus 1995).

Being a matched-filter approach, WMLE needs an accurate signal model to work well (parametric method). The other methods in this group are not as susceptible because these methods do not require the received signal shape to be known (nonparametric methods). In general, the shape of the received echo changes due to the changes in the center frequency and bandwidth in the signal for various effects including frequency-dependent attenuation and scattering, changes in the beam and different arrival time of the signals from different areas of the transducer except at the focus. In general, these changes are not precisely known.

Loupas and Gill (1994) analyzed the problem of 2D spectral analysis for discrete limited-duration signals. Loupas et al. (1995a, 1995b) introduced a 2D autocorrelation approach for the axial velocity estimation. The mean axial velocity is estimated from the estimates of both the Doppler and RF mean frequencies. Thus, this can potentially overcome the estimator bias due to frequency-dependent attenuation discussed by Ferrara et al. (1992). The 2D autocorrelator can be applied to both the complex envelope and the analytic RF signal. The results from simulation showed that 2D autocorrelator performs much better in the presence of modest velocity spread and high SNR; however, these enhancements become marginal as the condition degrades, which worsens the correlation between Doppler and RF fluctuations. The 2D autocorrelator and the cross-correlator are shown to be mathematically equivalent under a set of specific conditions. They were found to perform in an identical manner in high SNR conditions; however, the former was found to be more robust in low SNR conditions.

For presenting quantitative blood velocity information, Doppler spectrum analysis with a velocity-time display has proven to be very robust and accurate (Torp and Kristoffersen 1995). These authors presented a tracking method that suppresses frequency aliasing in the Doppler spectrum velocity-time display. They demonstrated that tracking in 2D transform space, as described by Wilson (1991), can be shown to be equivalent to a tracking in 2D (slow time-fast time) space.

Thus, several tracking methods have been developed. They have all been found to perform well. Some of these methods suffer from notable limitations, including aliasing effects in the 2D FFT and the necessity to know the received signal shape in the WMLE, and some of these methods are computationally intensive. However, the butterfly search can be implemented in parallel and with elementary digital operations.

We hope this discussion of tracking methods has helped to overcome the shortcomings of the review article and has provided a complementary perspective.

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#### IN RESPONSE TO S. K. ALAM AND K. J. PARKER

##### To the Editor-in-Chief:

I appreciate the opportunity to review and comment on the Letter to the Editor by Alam and Parker, which provides a perspective on the review article ‘‘Colour Flow Mapping.’’ Our article includes a broad overview of many aspects of color flow mapping instruments including clinical applications, limitations of current instruments, sources of error, system architecture, transducers, safety, color maps, contrast agents and several aspects of signal processing. With the suggested outline, diverse potential audience and proposed page budget, a detailed review of velocity estimation techniques was not possible. Given the very general nature of the article and the brief paragraph describing our velocity estimation technique within it, the detailed discussion of specific techniques in the Letter by Alam and Parker seems misplaced as a response to our article. Also, our paper was submitted in early February 1996 and accepted in March 1996. Thus, two of the three Alam and Parker articles, as well as the book by Jensen, had not yet been published when we submitted the paper. This does not imply that additional excellent articles could not be referenced in every area of the review. Indeed, this is probably the case for every review article currently in print.

The framework for the signal processing discussion in our article is based on the classical work of (Van Trees 1968), which has the advantage of providing general expressions for local and global error without subclassification for pulsed transmitted signals. This framework also avoids differentiation of time and frequency domain techniques. Because estimators such as the autocorrelator can be derived in either domain, this distinction seems less useful in our application. In addition, our article indicated that the Van

Trees formalism leads to guidelines for systems that may track red blood cells.

Beyond this, I certainly agree that the WMLE (Ferrara and Algazi 1989) is very similar to the butterfly technique (Alam and Parker 1995) and that it is always interesting to examine multiple paths for deriving signal processing strategies. It seems best to leave detailed analyses of signal processing techniques to the peer-reviewed literature, but I will briefly address points that could be misinterpreted in the Letter by Alam and Parker. The Letter described three ‘‘key’’ modifications required to achieve equivalence between the WMLE and the butterfly, and I refer to one reference in each case to answer these points.

- (1) The Letter reports that the WMLE involves a matched filter that must be changed to a rectangular window; however, both techniques similarly require the multiplication of the echo by an axial window, followed by integration over time. The WMLE simply provides a theoretical basis for the choice of the window length and shape. (Ferrara et al. 1993) used a rectangular axial window with a length matched to that of the received pulse from a point scatterer, although the technique is relatively insensitive to the axial window shape.
- (2) The Letter reports that the WMLE includes a higher-order velocity term; however, there is no higher-order velocity term. The term in each estimator that involves a  $v$  has the same meaning (Ferrara and Algazi 1991).
- (3) The Letter states that ‘‘following the publication of Alam and Parker’’ Ferrara and Algazi added normalization; however, Ferrara and Algazi (1995) hold a US patent on normalized likelihood, which was issued prior to publication of the articles by Alam and Parker.

In an effort to be brief, a minimal list of references has been used. A complete list of WMLE references will be avail-